VECTOR INTEGRATION

D_3 (H) 5^{th} Paper

Q. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where

$\vec{F} = x^2 y^2 \vec{i} + y\vec{j}$ and C is $y^2 = 4x$ in the xy-plane from (0,0) to (4,4).

Soln

In xy-plane, $z = 0$

$\therefore \vec{r} = x\vec{i} + y\vec{j} \Rightarrow d\vec{r} = dx\,\vec{i} + dy\,\vec{j}$

$\therefore \vec{F} \cdot d\vec{r} = (x^2 y^2 \vec{i} + y\vec{j}) \cdot (dx\,\vec{i} + dy\,\vec{j})$

$= x^2 y^2\, dx + y\, dy$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C (x^2 y^2\, dx + y\, dy) \quad \text{——— (1)}$$

Now, $y^2 = 4x \Rightarrow y\,dy = 2dx$

and limit is x from 0 to 4. so, eq(1) becomes

$$\Rightarrow \int_C \vec{F} \cdot d\vec{r} = \int_{x=0}^{4} (x^2 . 4x + 2)\, dx$$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = \int_{x=0}^{4} (4x^3+2)\,dx$$

$$= \left[x^4 + 2x\right]_0^4$$

$$= 4^4 + 2\times 4 = 256+8 = 264.$$

Q. Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = (x^2-y^2)\vec{i} + xy\vec{j}$ and c is the arc of $y = x^3$ from $(0,0)$ to $(2,8)$.

Soln. $\vec{F}\cdot d\vec{r} = (x^2-y^2)\,dx + xy\,dy \quad \text{---(1)}$

$$\because y = x^3 \Rightarrow dy = 3x^2\,dx$$

so (1) becomes

$$\vec{F}\cdot d\vec{r} = (x^2 - x^6)\,dx + x.x^3.\,3x^2\,dx$$

$$= (x^2\,dx + 2x^6\,dx)$$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = \int_{x=0}^{2} (x^2\,dx + 2x^6\,dx) = \left[\frac{x^3}{3} + \frac{2}{7}x^7\right]_0^2$$

$$= \frac{8}{3} + \frac{2}{7}\times 128 = \frac{824}{81}$$